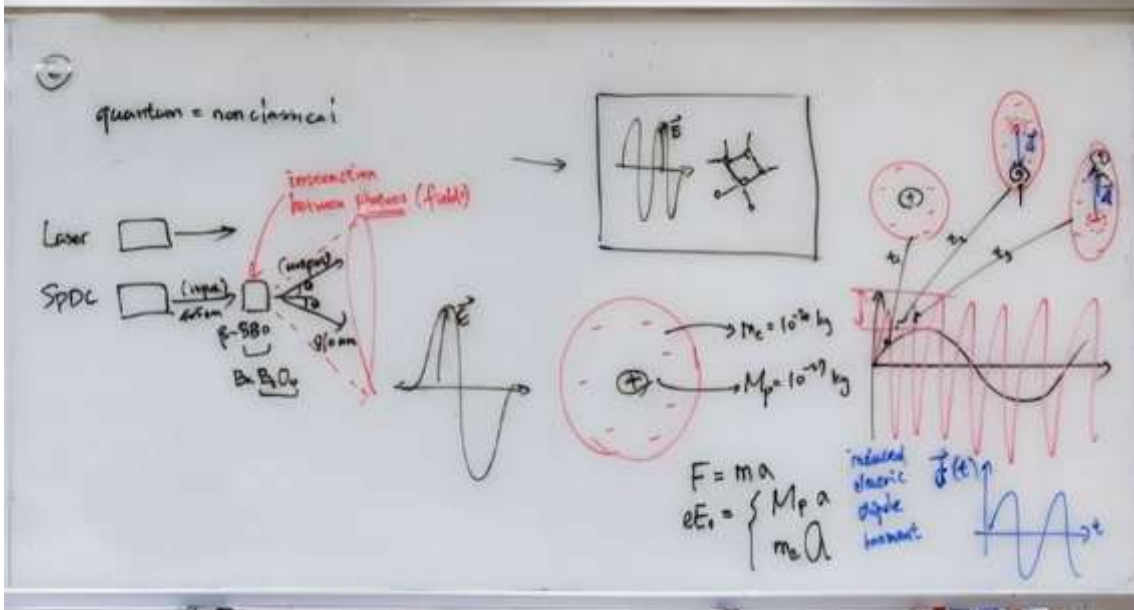
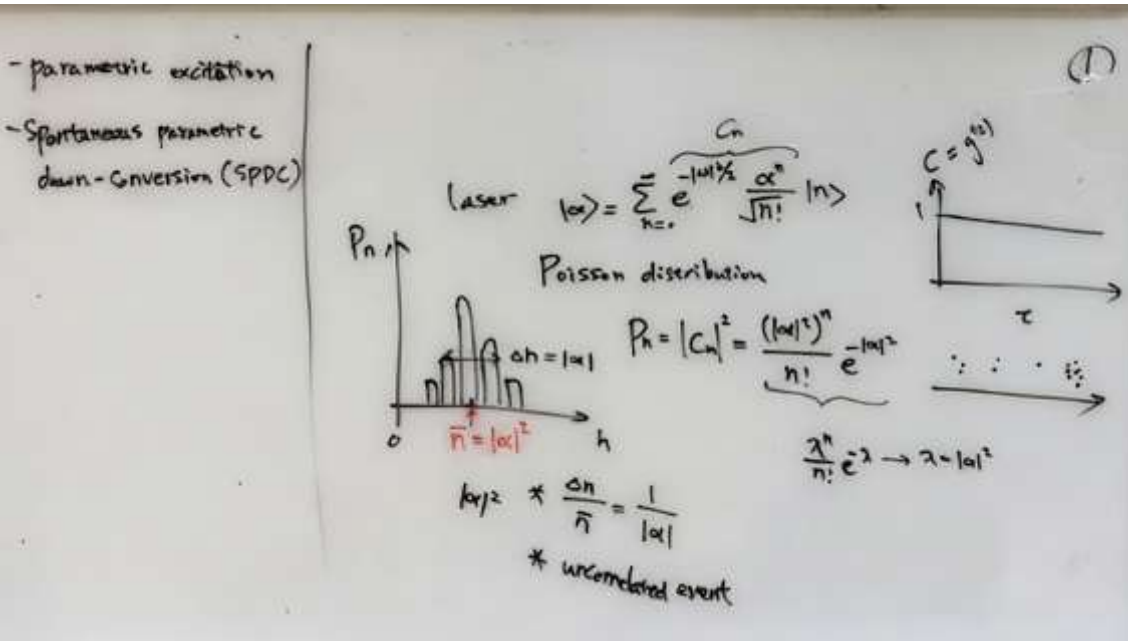




Lecture 8 (optics) SPDC mechanism





- parametric excitation
- Spontaneous parametric down-conversion (SPDC)

Maxwell's equation in material

mode function (intensity distribution)

plane wave: $e^{i\vec{k}\cdot\vec{r}}$

single-slit diffraction: $\frac{\sin(kr)}{kr}$

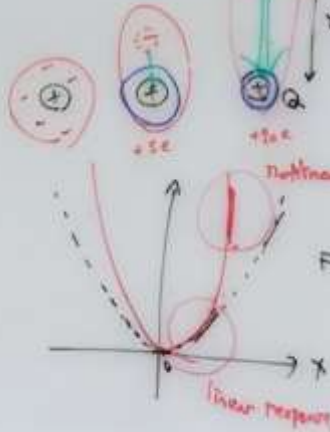
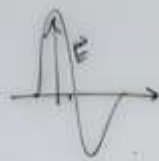
$$\text{vacuum} \quad \left\{ \begin{array}{l} \vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{E} = \rho/c \\ \vec{\nabla} \times \vec{B} = \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J} \\ \vec{\nabla} \cdot \vec{B} = 0 \end{array} \right.$$

atom density $\downarrow$

$$\text{material} \quad \left\{ \begin{array}{l} \vec{P} = N \vec{d} \\ \vec{M} = N \vec{m} \\ \vec{D} = \epsilon_0 \vec{E} + \vec{P} \\ \vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M} \end{array} \right.$$

$$\left\{ \begin{array}{l} \vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{D} = \rho/c \\ \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \\ \vec{\nabla} \cdot \vec{B} = 0 \end{array} \right.$$

(1)



$$d = Qx = f(E) = \epsilon^0 E + \epsilon^1 E^2 + \epsilon^2 E^3 + \dots$$

$$\vec{P} = \epsilon_0 (\chi^{(1)} \vec{E} + \chi^{(2)} \vec{E}^2 + \chi^{(3)} \vec{E}^3 + \dots)$$

dipole moment (susceptibility)

Vacuum permittivity (relative vacuum permittivity / dielectric constant)

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} = \epsilon_r^0 + \chi^{(1)} + \chi^{(2)} + \chi^{(3)} + \dots$$

$$P_i = \epsilon_0 \left(\chi_{ij}^{(1)} E_j + \chi_{ijk}^{(2)} E_j E_k + \chi_{ijkl}^{(3)} E_j E_k E_l + \dots \right)$$

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$





⑤ linear response $\vec{P} = \epsilon_0 \chi^{(1)} \vec{E}$, $\vec{M} = \frac{1}{\mu_0} \chi^{(2)} \vec{B}$

$$\vec{D} = \epsilon_0 \vec{E} + \epsilon_0 \chi^{(1)} \vec{E} = \epsilon_0 (1 + \chi^{(1)}) \vec{E} = \epsilon \vec{E}$$
$$\vec{H} = \frac{1}{\mu_0} \vec{B} - \frac{1}{\mu_0} \chi^{(2)} \vec{B} = \frac{1}{\mu_0} (1 - \chi^{(2)}) \vec{B} = \frac{1}{\mu} \vec{B}$$

linear-response

$$\begin{cases} \vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{E} = \rho/\epsilon \\ \vec{\nabla} \times \vec{B} = \mu (\vec{j} + \epsilon \frac{\partial \vec{E}}{\partial t}) \\ \vec{\nabla} \cdot \vec{B} = 0 \end{cases} \rightarrow \vec{\nabla}^2 \vec{E} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

nonlinear response $(\mu = \mu_0)$

$$\vec{\nabla}^2 \vec{E} - \mu_0 \frac{\partial^2}{\partial t^2} (\epsilon_0 \vec{E} + \vec{P}) = 0$$
$$\vec{P} = \epsilon_0 (\chi^{(1)} \vec{E} + \chi^{(2)} \vec{E}^2 + \dots)$$

$v = \frac{1}{\sqrt{\mu \epsilon}}$
 $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

Index of refraction $n = \frac{c}{v}$

$$\vec{\nabla}^2 \vec{E} - \mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \epsilon \chi^{(2)} \frac{\partial^2 \vec{E}^2}{\partial t^2}$$

interaction between fields

resonant evolution

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_p$$

parametric approximation $\vec{E}_p$ does not change by the interaction ($\vec{E}_p$ stays intact)

$$\begin{cases} \vec{\nabla}^2 \vec{E}_1 - \mu_0 \epsilon \frac{\partial^2 \vec{E}_1}{\partial t^2} = \mu_0 \epsilon \chi^{(2)} \frac{\partial^2 (\vec{E}_2 \vec{E}_p)}{\partial t^2} \\ \vec{\nabla}^2 \vec{E}_2 - \mu_0 \epsilon \frac{\partial^2 \vec{E}_2}{\partial t^2} = \mu_0 \epsilon \chi^{(2)} \frac{\partial^2 (\vec{E}_1 \vec{E}_p)}{\partial t^2} \end{cases}$$
$$\vec{E}_1 = e^{i\vec{k}_1 \cdot \vec{r}} e^{-i\omega_1 t} f_1(t)$$
$$\vec{E}_2 = e^{i\vec{k}_2 \cdot \vec{r}} e^{-i\omega_2 t} f_2(t)$$
$$\vec{E}_p = e^{i\vec{k}_p \cdot \vec{r}} e^{-i\omega_p t} f_p(t)$$
$$\vec{\nabla}^2 \vec{E}_1 = -k_1^2 \vec{E}_1 = -\frac{1}{\mu_0} \frac{\partial^2 \vec{E}_1}{\partial t^2} = \mu_0 \epsilon \chi^{(2)} \frac{\partial^2 (\vec{E}_2 \vec{E}_p)}{\partial t^2}$$
$$\vec{\nabla}^2 \vec{E}_2 = -k_2^2 \vec{E}_2 = -\frac{1}{\mu_0} \frac{\partial^2 \vec{E}_2}{\partial t^2} = \mu_0 \epsilon \chi^{(2)} \frac{\partial^2 (\vec{E}_1 \vec{E}_p)}{\partial t^2}$$

resonance

parametric decay

$$\cos(\omega_1 t) \cos(\omega_2 t) = \frac{1}{2} [\cos((\omega_1 + \omega_2)t) + \cos((\omega_1 - \omega_2)t)]$$
$$\frac{1}{2} [\cos((\omega_1 + \omega_2)t) + \cos((\omega_1 - \omega_2)t)]$$

linear response

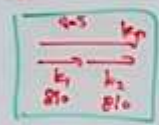




- ✓ parametric excitation
- Spontaneous parametric down-conversion (SPDC)

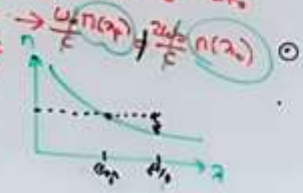


$$\begin{cases} \hbar\omega_p = \hbar\omega_1 + \hbar\omega_2 & \text{energy conservation} \\ \hbar\vec{k}_p = \hbar\vec{k}_1 + \hbar\vec{k}_2 & \text{momentum conservation} \end{cases}$$

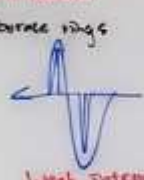
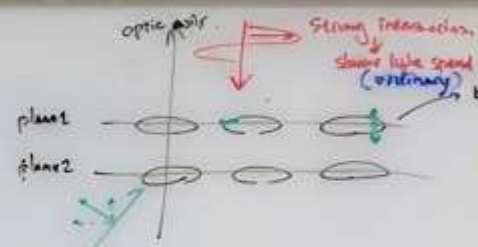
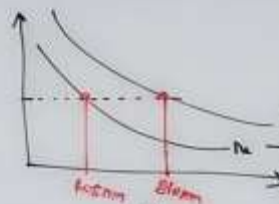


$$\begin{aligned} \omega_p &= \omega_1 + \omega_2 = 2\omega_0 \\ \vec{k}_p &= \vec{k}_1 + \vec{k}_2 = 2\vec{k}_0 \end{aligned}$$

Sellmeier equation: $n = \sqrt{A + \frac{B}{\lambda^2 - C} - D\lambda^2}$



p-BBO $\rightarrow$ birefringence material
 $n_o(\lambda), n_e(\lambda)$



Strong Interaction: slower light speed in material (ordinary)
Weak Interaction: faster light speed in material (extraordinary)



$$\begin{aligned} \frac{1}{n_p^2} &= \frac{\cos^2 \theta_p}{n_o^2(\lambda_p)} + \frac{\sin^2 \theta_p}{n_e^2(\lambda_p)} \\ n_1 &= n_o(\lambda_1) \\ n_2 &= n_o(\lambda_2) \end{aligned}$$



$$\begin{cases} \omega_p = \omega_1 + \omega_2 \\ k_p = \cos \theta_1 k_1 + \cos \theta_2 k_2 \\ \sin \theta_1 k_1 = \sin \theta_2 k_2 \end{cases}$$



